BOUGUER CORRECTION FOR A SPHERICAL EARTH: APPLICATION TO THE ETNA DATA

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Abstract

The procedures to perform complete Bouguer corrections are a critical concern mainly when rugged topographies and large investigation areas are interested. These corrections generally include the simple slab and terrain corrections. In recent years it was recognized that a curvature correction is necessary also for exploration surveys. In order to standardize the correction parameters and to allow the joining together of different data sets, the choice of an outer radius of 166.7 km was suggested for the complete Bouguer correction. This paper presents an automatic procedure to perform the Bouguer correction considering as reference the spherical surface through each gravity station. The terrain correction in the “outer zone” (from 2.5 km to 166.7 km) is performed calculating, from gridded average elevation data, triangular polyhedra with a face on the reference sphere. For the “inner zone” (station to 2.5 km) triangular polyhedra are constructed by modelling the topographic surface with a triangular faceted surface resulting from the Delaunay triangulation of the digitized elevation data. An example of the application of the proposed procedure to the Etna gravimetric data is shown.

1. Introduction

As it is well known the Bouguer anomaly represents the residual values between the observed gravity and the gravity components generated by a simple earth model (the ideal earth model). As stated by Chapin (1996) the gravity corrections, adopted for obtaining the Bouguer anomalies, “attempt to make up for the incorrect assumptions made in the original earth model”. It was lengthy debated in the last ten years in the scientific community on the procedures to carry out the corrections for prospecting targets, mainly as regards the earth model to adopt for the simple Bouguer correction (a planar model or a spherical one) and the limit to which the terrain corrections should be applied [LaFehr, 1991a, 1991b, 1998; Talwani, 1998]. Following LaFehr (1998), if major changes in elevation occur between stations also for local surveys a spherical cap with surface radius of 166.7 km from a station must be considered both for the simple Bouguer correction and the terrain one. The selection of this distance, although arbitrary, is justified by the need of standardization in the correction procedures. As a consequence of this choice it is (1) avoided the introduction of fictitious elevation-dependent anomalies and (2) eliminated the occurrence of misties between independently conducted adjacent or overlapping surveys [LaFehr, 1991a, 1998].

We consider the argumentations of LaFehr well founded and his standardization requirement to be observed for gravity surveys in a large part of the Italian territory.

In this paper we present a procedure for the complete Bouguer correction with an example of application to the Etna field data.

2. Complete Bouguer correction

The classical complete Bouguer correction is a three-step procedure [Bullard, 1936]:

- Simple Bouguer correction (Bullard A) – it is an elevation correction where the air in the Free-air correction is replaced by an infinite slab filled with rock.
- Curvature correction (Bullard B) – it attempts to correct the physically unaccountable slab model of the earth making its shape more realistic; it is not generally used.
- Terrain correction (Bullard C) – it accounts for the topography around the station out to the maximum distance assumed to produce a meaningful contribution to the data values.

2.1. Bullard A+B correction

The simple Bouguer correction and the curvature one can be unified in a single step considering directly a spherical cap. Fig.1 shows, in a meridian plane, a sketch of the topographic surface and the spherical cap around a station S. See LaFehr (1991b) for the algorithms to calculate the attraction of a spherical cap.
2.2. Terrain correction

The procedure to correct gravimetric data for the topographic relief around the stations is the processing step more susceptible to introduce errors in the computed anomalies. This accounts for the large number of papers on this subject published in the last several decades.

Considering, for simplicity, only the completely automated methods, the difference among them is the way the topographic surface is mathematically represented.

The starting point is a digital representation of the topographic surface. The digital earth models (DEM or DTM) represent the surface at the nodes of a regular grid through elevation or mean values. The used gridded data come from databases, when available, or are specially constructed for a study area.

Considering the inverse square distance law for gravitational attraction, an increasing accuracy in the surface approximation is necessary, the distance from the station decreasing. To take into account this requirement, moving out from the station the area is generally divided into different zones in which different strategies are adopted for the correction. Thus, an inner or near zone and an outer or distant zone are generally considered. In some cases one or more intermediate zones are introduced between the previous ones [Blais and Ferland, 1984; Ma and Watts, 1994].

Fundamentally, for the inner zone, the gridded data are fitted with a mathematical surface – a set of Gaussian functions [Herrera-Barrientos et al. 1991], a multiquadric equation [Krohn, 1976], a triangle-based surface [Cogbill, 1990] – and interpolated values computed at a more dense grid points distribution [Blais and Ferland, 1984] or mean values calculated in circular sectors similar to the Hammer’s chart [Herrera-Barrientos and Fernandez, 1991]. Once this is done, corrections are calculated using different approximations of the surface: flat-top prisms [Hammer, 1939], inclined-top circular prisms [Olivier and Simard, 1981], dipping triangular elements [Zhou et al., 1990].

While for the distant zones a regular structure of the data could be justified by data storage needs in the framework of databases covering large areas (at a regional, state or global scale), the complex procedures adopted to represent the topography in the area nearest a station seem to us unjustified for the nowadays computer memory and power capabilities.

Our reasoning is the following. The original digital terrain data are produced from maps or aerial photographs by manual digitizing, semi-automatic line following or automatic raster scanning or provided by digital satellite imageries. Whatever be the method to generate surfaces from the data, the accuracy of the reconstruction cannot be better than the original data. Any other processing step applied to the data is likely...
to misrepresent the terrain model and to introduce errors in the terrain correction. Besides, with reference to the application of the surface modelling to gravity data correction, we must consider that hardly the position of the gravity stations coincide with the nodes of a DTM, requiring a supplemental approach to introduce them into computation [Ma and Watts, 1994]. Hence, a method directly using the irregular distribution of the original data is, to our opinion, more suitable to perform, at the maximum accuracy contained into the elevation data and in a simpler way, the modelling of the topography.

Triangulated Irregular Networks (TINs) represent a data structure that directly relies on data. Terrain models are consequently represented by triangular faceted surfaces. Triangulated surface models are the most widely used in many different application domains, including computer graphics, geographic data processing, computer vision and computer aided design. The advantage of this method is the possibility of including surface features and on the simplicity of the topological structure. In the framework of digital terrain modelling, triangle-based models allow for the variable resolution connected with different topography behaviours. Thus, TINs succeed in representing a surface at a certain level of accuracy using a smaller amount of data.

Among the methods for triangulating a set of irregularly spaced points in 2-D and 3-D spaces, the Delaunay one satisfies some optimality criteria. In particular, the method generates triangles that are as much equiangular as possible [Lawson, 1977; Preparata and Shamos, 1985], thus avoiding thin and elongated triangular facets, it minimizes the maximum circumcircle [D’Azvedo and Simpson, 1989] and the maximum contained circle [Rajan, 1994] (the last two conditions can be equivalently expressed as follows: considering any four points, such points do not belong to the same circle).

Using the triangulation method, the terrain correction can be carried out through triangular prisms [see, for example Zhou et al. 1990].

3. Terrain correction method

Fig. 2 shows the scheme adopted for the terrain correction. The inner zone is defined as the 5 km² square region centred at each station. In this zone the height field is approximated from digitized 1:25000 topographic maps through Delaunay triangulations.
There are many Delaunay triangulations algorithms with different computer speed and memory requirements [see, for example: Barber et al. 1996; Bourke, 1989; Renka, 1984; Shewchuk, 1996, 1997, 2002]. Among the freely available computer routines we have preferred the Triangle program by Shewchuk (1996) for its efficiency and computational speed. Fig. 3 shows an example of digitized points and the resulting triangulations. For drawing clarity it is represented only the 1 km\(^2\) area surrounding the station.

![Fig. 3 - Method used to approximate the topographic surface in the inner zone; a) example of digitized elevation points around staz1 and b) resulting Delaunay triangulations.](image)

In the distant zone, between the inner one and the circular, 170 km radius, outer edge the topography is approximated by 7.5”x 10” gridded data representing mean elevation values. The gravity effect of the topography both in the inner and outer zones is evaluated considering triangular polyhedra. The relationships for homogeneous polyhedral bodies by Okabe (1979) are used. The faces of the polyhedra are the triangular facets in the inner zone and three neighbouring grid values in the outer zone, on the topographic surface, and their geocentric projections on the reference sphere, respectively. To apply the Okabe’s formulae transformations from ellipsoidal to plane coordinates are needed. In particular, from figure 1, assuming the local sphere at each station to approximate the ellipsoid, a station \(S\), a generic point \(P\) on the surface and its projection on the sphere \(U\), have ellipsoidal coordinates \(S(r_0, \phi_0, \lambda_0)\), \(P(r, \phi, \lambda)\) and \(U(r_0, \phi, \lambda)\), respectively; where \(OT=OV=R_0\) is the mean earth radius, \(TS=h_0\) and \(VP=h\) are the heights above the mean sea level of the station and the point, respectively, \(r_0=R_0+h_0\) and \(r=R_0+h\). \(\phi_0\) and \(\phi\) are the ellipsoidal latitudes and \(\lambda_0\) and \(\lambda\) are the ellipsoidal longitudes.
the ellipsoidal longitudes of the two points, respectively. For simplicity, Fig. 1 shows the points S and P in the same meridian plane.

Considering a Cartesian coordinate system with origin in S, the plane $XY$ tangent to the local sphere in S, $X$ axis northward, $Y$ axis eastward and $Z$ axis toward the centre of the sphere, the coordinates of the points P and U become:

\[
\begin{align*}
X &= -r \cos \varphi \sin \varphi_0 \cos(\lambda - \lambda_0) + r \sin \varphi \cos \varphi_0 \\
Y &= r \cos \varphi \sin(\lambda - \lambda_0) \\
Z &= -r \cos \varphi \cos \varphi_0 \cos(\lambda - \lambda_0) - r \sin \varphi \sin \varphi_0 + r_0
\end{align*}
\]

\[
\begin{align*}
X_1 &= -r_0 \cos \varphi \sin \varphi_0 \cos(\lambda - \lambda_0) + r_0 \sin \varphi \cos \varphi_0 \\
Y_1 &= r_0 \cos \varphi \sin(\lambda - \lambda_0) \\
Z_1 &= -r_0 \cos \varphi \cos \varphi_0 \cos(\lambda - \lambda_0) - r_0 \sin \varphi \sin \varphi_0 + r_0
\end{align*}
\]

4. Application to Etna data

The method previously described was used for reprocessing the gravity data surveyed for the study of the Mt. Etna volcano [Loddo et al. 1989]. The Bouguer anomaly shown in the cited paper was obtained performing the topographic correction with a manual chart method over a distance of 28 km from each station and using the 1930 International Gravity Formula. For the new processing step the original data were standardized using the IGSN71 system [Morelli et al., 1974] and the Italian First Order Gravity Network [Marson and Morelli, 1978]. Besides, the Geodetic Reference System 1980 [Moritz, 1984] was applied and the data corrected for the spherical cap and the topography out to 166.7 km. In order to numerically compare the old and the new processing procedures, the infinite slab and the old topographic corrections were applied to the transformed data.

Fig. 4 shows the approximate area over which the topographic corrections of all the Etna data were carried out. The Sicily mountain chains are completely contained in the area together with a large part of the calabrian Apennines. Besides, large sea-covered surfaces are present. For these last zones, the 7.5”x10” mean elevation gridded data were computed by digitizing the bathymetric maps. The obtained database was connected to the existing one for inland zones.

Fig. 4 - Approximate extent of the 167 km radius area considered for the computation of the terrain effect on the Etna data.
Considering the elevation values of the Etna stations, the maximum difference between the infinite slab and the spherical cap amounts to about 2-3 mGal. On the contrary, differences larger than 10 mGal result for the effect of the topography as figs. 5 and 6 indicate. The maps are constructed using a mass density of 2650 kg/m$^3$.

![Fig. 5 - Topographic effect for the Etna stations evaluated with the manual chart method out to 28.8 km from each station. Contour lines in mGal.](image)

![Fig. 6 - Topographic effect for the same stations of Fig. 5, computed through our processing method. Contour lines in mGal.](image)
The differences between the complete Bouguer corrections obtained with the new processing method and the older one are mapped in fig. 7. As it was expected, the differences increase with the elevation resulting in substantial changes in the Bouguer anomalies.

![Fig. 7 - Difference in mGal between the complete Bouguer corrections for the Etna data resulting from the old and the new processing method.](image)

To our knowledge, there are few papers that take into account the topographic effect out to 170 km [Krohn, 1976; Banerjee, 1998] and just one that shows, with a field example, the different contributions to the corrections by the topography in the first 20 km range and by the rest of the 170 km range area [Banerjee, 1998]. The survey presented in the last cited paper, carried out in an area with very large elevation changes, shows results similar to ours as regards the contribution of the elevation changes to the terrain correction. In particular, it was confirmed the increase with the elevation rising both of the terrain correction and of the contribution to the total terrain correction, by the region between 20 km and 170 km from the gravity station.

5. Concluding remarks

We have presented an automatic method to perform the complete Bouguer correction on a spherical earth considering the effect of the topography out to 170 km from the stations. For the zone nearest the station (the inner zone), where the topography have the largest influence on the accuracy of the corrections,
the earth surface is approximated by a TIN structure generated by digitized height values. The method is simple and does not introduce further approximations to the original elevation data.

Our results show that the contribution of the topographic correction to Bouguer anomalies changes with the elevation of the station resulting in a different effect over short distances. Besides, the influence of the distant topography cannot be ignored when large changes in elevation among stations exist. As standard the processing steps to obtain the Bouguer anomalies should include the effect of the earth curvature and of the terrain out to 170 km from the station.

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REFERENCES


